DOCUMENT RESUME

ED 156 708 95 IR 007 167

AUTHOR Swinton, Spencer S.; And Others
TITLE The Role of Short-Term Memory in the Development of Logical Operations Skills. Technical Report No. 445.
INSTITUTION Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning.
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.
REPORT NO WRDCCL-TR-445
PUB DATE Dec 77
CONTRACT OB-NIE-G-78-0117
NOTE 45p.

EDRS PRICE MF-$0.83 HC-$2.06 Plus Postage.
DESCRIPTORS Auditory Perception; Classification; *Cognitive Development; Cognitive Measurement; *Cognitive Processes; Correlation; Developmental Stages; Elementary Education; *Logical Thinking; Longitudinal Studies; Mathematical Models; *Memory; *Performance Factors; Performance Tests; *Predictor Variables; Statistical Analysis; Task Analysis; Task Performance; Visual Perception
IDENTIFIERS Cross Lagged Panel Technique; *Piagetian Tasks

ABSTRACT

The longitudinal interrelations among age, visual and auditory short-term memory, and the concrete operational tasks of class inclusion and combinatorial reasoning were investigated over four testing occasions in a sample of 134 students, initially ranging from 5 to 12 years in age. The Piagetian task battery involved placement of colored triangles into superordinate, intersecting, and subclasses, and the systematic pairing of colored chips. Auditory sequential memory, memory span, and visual orientation memory tests involved digit series, word series, and two-dimensional geometric figures in varying spatial orientations. Year four logical operations performances were predicted from earlier variables by means of multiple regression analysis. Memory variables were significant predictors of later logical task performance in all regressions except in predicting final combinatorial reasoning status from year two data. Application of two models for the attribution of cause from correlational evidence indicated that combinatorial reasoning is a better predictor of short-term visual memory changes than the memory task is a predictor of combinatorial reasoning development. Visual memory should not, therefore, be regarded as a source of performance variance to be controlled in the investigation of logical operations development, but it should be seen as an outcome and a precondition for such cognitive restructuring. (Author/JAC)

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THE ROLE OF SHORT-TERM MEMORY IN THE DEVELOPMENT OF LOGICAL OPERATIONS SKILLS

by Spencer S. Swinton, Thomas S. Sipple, Frank H. Hooper, and Craig L. Hougum

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WISCONSIN RESEARCH AND DEVELOPMENT CENTER FOR COGNITIVE LEARNING

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Spencer S. Swinton, Thomas S. Sipple,
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Report from the Project on
Studies of Instructional Programming for the Individual Student

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December 1977

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the National Institute of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education and no official endorsement by that agency should be inferred.

Center Contract No. OB-NIE-G-78-0117

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The Wisconsin Research and Development Center is supported with funds from the National Institute of Education and the University of Wisconsin.

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ACKNOWLEDGEMENTS

The authors wish to thank Dr. Charles Brainerd of the University of Western Ontario for consulting on theoretical matters throughout the development of the study. We also wish to extend our gratitude for their cooperation to Lyman Olson, Administrative Assistant; Virginia Carlson, Ralph Crowe, Arthur Engel, Robert Harris, Merlin Kenitzer, W. W. Kirchoff, George Kolak, Ralph Napper, Merlin Nundahl, and Peter Peterson, elementary school principals; Laverne Larsen and John Smiley, junior high school principals; and the staff and the students of Royce, Waterman, Merrill, Cunningham, Converse, McLenegan, Todd, Gaston, Burdge, and McNeel Elementary Schools and Roosevelt and Lincoln Junior High Schools of the Beloit, Wisconsin Public School System. We wish to acknowledge the diligent efforts of Susan Anderson, Nikki DeFrain, Roberta Dihoff, Arthur Gonchar, Patricia Cone-Gorham, Rosie McMahan, Priscilla Roth, Richard Stern, and Tom Toniolo in administering the tasks.

An abridged version of this report was presented at the Biennial Meeting of the Society for Research in Child Development, New Orleans, La., March, 1977.

TABLE OF CONTENTS

		Page
	Acknowledgements	iv
	List of Tables	vii
	Abstract	ix
I	Introduction	1
II	Method	3
III	Results and Discussion	7
	References	23-24
	Appendix	27

LIST OF TABLES

Table		Page
1	Sampling Design for the Longitudinal Study	6
2	Means and Standard Deviations of Chronological Age and the Five Measures in Each of the Four Assessment Years	10
3	Intercorrelations Among the Measures for the Four Assessment Years	11
4	Prediction Equations: Fourth Year Piagetian Task Performances from Years 1 to 4 Variables, With and Without Age as a Predictor	13
5	Cross-lagged Correlations: Visual Memory (VM) and Class Inclusion (CI)	17
6	Cross-lagged Correlations: Visual Memory (VM) and Combinatorial Reasoning (CR)	18
7	Cross-lagged Correlations: Word Memory, Digit Memory, and the Piagetian Measures	19
8	Four-Wave Causal Equations	22

ABSTRACT

Inferring relationships among unobservable logical competencies from performance on specific tasks embodying these competencies is complicated by the fact that any such task involves other sources of performance variance: e.g., memory load, stimulus saliency, vocabulary demands, as well as its underlying cognitive structure. As Niemark has pointed out, "It is impossible to determine from only a single measure whether one is dealing with general competence or with situation-specific performance [Niemark, 1975, p. 573]."

The study reported here attempted to statistically control the influence of one source of performance variance, short-term memory, to better describe the relationships among logical tasks due to other factors, including shared logical components.

The longitudinal interrelations among age, visual and auditory short-term memory, and the concrete operations period logical tasks of class inclusion and combinatorial reasoning were investigated over four testing occasions in a sample of 134 children, initially ranging from 5 to 12 years in age.

Year four logical operations performances were predicted from eariler variables by means of multiple regression, yielding multiple correlations ranging in magnitude from .412 to .598. Memory variables were significant predictors of later logical task performance in all regressions except in that predicting final combinatorial reasoning status from year two data.

Two models (Kenny, 1973, 1975) were fit to the cross-lagged panel correlations resulting from the study. Both sets of assumptions led to the conclusion that one of the logical tasks, combinatorial reasoning, is a better predictor of short-term visual memory changes than the memory task is a predictor of combinatorial reasoning development.

These results suggest that visual memory should not simply be regarded as a source of performance or "nuisance" variance to be controlled in the investigation of logical operations development, but must be seen as an outcome rather than only a precondition for such cognitive restructuring.

I

INTRODUCTION

It is obvious that inferences concerning the synchrony or asynchrony of developmental emergence of Piagetian logical operations structures are ultimately based upon subjects' performances on tasks designed to embody such cognitive structures. As we have emphasized previously, orthodox Piagetian theory predicts a tight interrelationship among structures and tasks characteristic of a given developmental period or stage, e.g, concrete operations, and a sequential or predictive relationship between tasks derived from adjacent developmental periods, i.e., concrete and formal operations (Hooper, Swinton, & Sipple, 1976). This follows from the basic tenet that, "Inherent in Piaget's general system is the view that the logical structure of mathematical and physical systems is paralleled by the order of psychological (logical thought) development [Beilin, 1971, p. 116]."

Performance on any series of logical reasoning tasks, however, is apt to reflect sources of variability other than based upon the presumed underlying cognitive structure, such as stimulus salience, instructional set, response and attentional demands, and short-term memory requirements (See Flavell, 1977, pages 220-235 for a recent discussion of these diagnosis and related causal inference problems). The present report, based on a four year longitudinal study of children's development in logical operations and short-term memory behavior (Hooper & Klausmeier, 1973), examines the relationships among two measures of auditory memory span, visual orientation memory, and a representative index of the concrete operations period

(class inclusion) and of the formal operations period (combinatorial reasoning).

It was hypothesized that, in view of the surface task demands, individual differences in auditory memory span would predict a significant proportion of the variance of the class inclusion task, while visual memory would be more strongly related to combinatorial reasoning. If this were the case, it was argued that these performance aspects of the tasks should be taken into account in drawing inferences to underlying competence. In particular, relationships due to shared operational structures could be more accurately reflected by examining partial correlations, with the influence of individual differences in memory and the differing memory demands of the tasks statistically controlled.

If it proved to be the case that shared variance among logical operations tasks is due, to a measurable extent, to shared perceptual demands of these tasks, one information-processing approach would lead the investigator to discount that portion of the relationship as "performance variance," and to estimate the degree of structural similarity of logical operations behaviors only from that portion of the relationship not so "explained." However, this approach depends on the view that memory is a passive capacity, the development of which causes the emergence of logical structures (McLaughlin, 1963). More recent, dynamic views of memory development suggest that emerging logical structures may themselves contribute to the development of memoric skills (e.g., Pascual-Leone, 1970; Norman & Rumelhart, 1975). Under this approach, it would clearly not be appropriate to partial out memory as a common cause of a spurious

component of the relationships under study, since memory growth, to some extent, would be a result of this development. Two cross-lagged correlation models are applied to the data to investigate the degree to which causation might be viewed as reciprocal, rather than as flowing strictly from perceptual to logical operation development. It may be seen that although based on differing assumptions, the causal analyses yield similar results in these data.

II

METHOD

SUBJECT SAMPLE

One hundred and eighty school children enrolled in the Beloit, Wisconsin, public school system participated in the first-year assessments in the spring of 1973. Table 1 presents the general sampling design for the four-year longitudinal study. The children were randomly drawn from kindergarten, third, and sixth grade classrooms of four different elementary schools. These schools were selected in accord with the representative socioeconomic, ethnic, and general demographic classifications found in the city of Beloit.

Of the 180 children who took part in the first year of the study, 134 children were available for longitudinal assessment three years later in 1976. Attrition was primarily due to entry into parochial schools or to moving out of the city. In addition, control groups of children were assessed in the spring of 1974 and 1975 in order to determine the possible effects of repeated testing. Biases related to selective participation or repeated testings were notably absent from the present results (see Hooper, Swinton, & Sipple, 1976, pages 33-34).

TASK DESCRIPTIONS

Class Inclusion (score range = 0-5)

a. In reference to triangles, three red and two blue, the S is asked whether there are more members of the superordinate class (triangles) than

Table 1

SAMPLING DESIGN FOR THE LONGITUDINAL STUDY*

(Underlined entries are approximate mean ages and arrows represent continuity of subgroups)

	Time of Measurement			
Year of Birth	1973	1974	1975	1976
1967	6 (N = 60) [Kindergarten] →	7 (N = 48) →	8 (N = 46) →	9 (N = 41)
1967		7 (N = 24)		
1967			8 (N = 20)	
1964	9 (N = 60) [Third Grade] →	10 (N = 54) →	11 (N = 50) →	12 (N = 47)
1964		10 (N = 24)		
1964			11 (N = 22)	
1961	12 (N = 60) [Sixth Grade] →	13 (N = 54) →	14 (N = 52) →	15 (N = 46)
1961		13 (N = 24)		
1961			14 (N = 24)	

*Adapted from: Hooper, Swinton, & Sipple (1976), page 11.

members of the nested subclass (red figures). 1 trial. [The above procedure is repeated using circles in another trial.]

b. In reference to triangles, three red and two blue, and four blue circles, the S is asked whether there are more members of the superordinate class (triangles) than members of the nested subclass (red figure). 1 trial. [The procedure is repeated in another trial with the same stimuli using blue figures as the superordinate class and circles as the nested subclass.]

c. In reference to the same stimuli used in (b), the S is asked which of the intersecting classes (blue figures or triangles) is larger. 1 trial. 5 total trials. Adapted from Kofsky (1966).

Combinatorial Reasoning (score range = 0-28)

The S is requested to form systematically all possible (28) pairs of different colored chips without repeated pairs from eight different colors. Adapted from Goodnow (1962).

Memory Tasks

Three measures of memory ability were employed in the present assessment research. The auditory sequential memory test (Wepman & Morency, 1973b) consisted of forward digit series (2 to 8 digits) with two sets of differing numbers at each level (score range = 0-70). The auditory memory span test (Wepman & Morency, 1973a) consisted of forward word series with three trials of differing word sets at each level from two to six words (score range 0-60). The visual orientation memory test (Swinton, 1973) assessed the child's ability to recognize and identify (following a 5-second study interval) a two dimensional geometric figure (line drawing) from a pictorial array of the same figure in varying spatial orientations. There were four

possible choices for each of 20 items (score range 0-20).

Complete discriptions and a summary of the psychometric characteristics of the entire Piagetian task battery used in the present normative assessments are presented in Hooper, Brainerd, & Sipple (1975).

ANALYSES

In addition to the means and standard deviations, forward stepwise multiple regression predictions of each year's Piagetian performance from each previous year's data were obtained, and (partial) regression weights compared. In addition, the technique of cross-lagged panel correlation (Kenny, 1973) was applied to test the direction of causality of changes in memory and Piagetian performances in adjacent and nonadjacent years.

III

RESULTS AND DISCUSSION

Over the three-year period, the most stable instruments were the auditory memory tasks (r = .71, word and .66, digit) followed by visual memory (r = .55) and combinatorial reasoning and class inclusion (r = .41 and .35 respectively). The best single predictor of Piagetian performance three years later was initial status on the visual memory task, yielding slightly, but not significantly greater predictive power than did initial status on the Piagetian performances themselves. Final year combinatorial reasoning performance shared more variance with earlier memory tasks than did class inclusion, but both tasks initially appeared more related to this visual memory task than to auditory span.

The means and standard deviations of the present measures (including chronological age for all grade-levels combined) are presented in Table 2. Analyses of variance for the initial two assessment years (Hooper, Swinton, & Sipple, 1976) and the overall four year interval (Hooper, Sipple, & Swinton, in preparation), indicated significant main effects for grade levels and assessment years for all the measures. It should be noted that the present correlation analyses report data for the combined grades (N = 134).

The intercorrelations among the various measures for the time intervals are shown in Table 3. While all variables show significant correlations with age, combinatorial reasoning and visual memory in year one are highly age-related, while digit memory in year two and year four class inclusion exhibit

Table 2

MEANS AND STANDARD DEVIATIONS OF CHRONOLOGICAL AGE AND THE FIVE MEASURES IN EACH OF THE FOUR ASSESSMENT YEARS

Assessment Year	Age in months		Combinatorial Reasoning		Class Inclusion		Word Memory		Digit Memory		Visual Memory	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
Year one	107.66	29.59	17.34	10.05	3.57	1.40	29.94	9.88	29.11	12.02	13.37	4.40
Year two			21.55	7.82	3.98	1.23	36.13	10.75	33.72	13.94	15.42	3.69
Year three			23.12	7.09	4.31	1.00	37.96	11.39	36.23	14.90	16.44	3.14
Year four			25.15	5.57	4.52	.88	38.72	9.73	40.57	14.00	18.04	2.11

Table 3

INTERCORRELATIONS AMONG THE MEASURES
FOR THE FOUR ASSESSMENT YEARS

		Year one						Year two					Year three					Year four				
		Age	CR	CI	WM	DM	VM	CR	CI	WM	DM	VM	CR	CI	WM	DM	VM	CR	CI	WM	DM	VM
Year one	CR	.63																				
	CI	.47	.49																			
	WM	.41	.40	.36																		
	DM	.39	.36	.29	.70																	
	VM	.76	.64	.47	.39	.37																
Year two	CR	.43	.41	.35	.27	.21	.39															
	CI	.40	.38	.44	.24	.28	.39	.30														
	WM	.37	.23	.31	.55	.52	.34	.22	.30													
	DM	.20	.14	.19	.42	.50	.17	.33	.34	.64												
	VM	.57	.47	.38	.27	.31	.65	.43	.39	.30	.32											
Year three	CR	.39	.43	.31	.28	.17	.42	.39	.34	.02	.13	.24										
	CI	.29	.29	.32	.29	.16	.27	.32	.44	.24	.22	.40	.22									
	WM	.51	.34	.23	.67	.65	.43	.33	.36	.62	.56	.39	.30	.34								
	DM	.42	.32	.28	.58	.63	.39	.37	.31	.57	.65	.39	.24	.26	.74							
	VM	.46	.33	.28	.28	.22	.58	.32	.42	.25	.27	.59	.38	.36	.37	.37						
Year four	CR	.43	.41	.22	.33	.28	.46	.42	.43	.27	.28	.35	.47	.34	.41	.38	.37					
	CI	.19	.25	.35	.20	.22	.36	.14	.41	.20	.21	.34	.20	.38	.26	.33	.29	.45				
	WM	.30	.35	.38	.71	.71	.35	.22	.20	.63	.59	.35	.20	.22	.68	.65	.24	.30	.31			
	DM	.50	.47	.33	.64	.66	.51	.39	.32	.56	.50	.46	.32	.22	.70	.74	.44	.39	.33	.69		
	VM	.44	.29	.32	.30	.31	.55	.28	.41	.28	.18	.48	.41	.32	.34	.24	.61	.37	.29	.25	.35	

weak dependence on age ($r < .20$). It should be kept in mind that the age span of the sample, initially 5 to 12 years, and ultimately 8 to 15, inflates correlations with age. For example, while the correlation of year four combinatorial reasoning with age is .43, the nearly 5 to 1 ratio of standard deviations results in a raw regression coefficient of combinatorial reasoning on age of only .08: that is, it requires about a one year difference in age to result in a one-point predicted increment on the year four combinatorial reasoning task.

To allow for the possibility that relationships among tasks might be artifacts of mutual age-dependence, age was allowed to enter the regression equations used to predict final status on the two reasoning tasks from previous years' levels on reasoning and memory tasks. As Table 4 reveals, the inclusion of the age variable inflates the regression weights for the other variables in predicting class inclusion, but acting as a suppressor variable deflates the impact of other predictors of combinatorial reasoning. In none of the predictive regression equations, however, does the addition of age cause the contribution of the other variables to fall below significance. On the other hand, in the case of predicting final combinatorial reasoning from year one status, the addition of auditory and visual memory, the only two significant task variables, raises the multiple r only .06 from the correlation (.43) obtained from age alone, suggesting that much of the variance in the system can be explained by age-dependence.

Examining the standardized regression weights of the variables from each year entering significantly into the prediction of final level on reasoning tasks, and the resulting multiple correlations, it is apparent

TABLE 4

PREDICTION EQUATIONS: FOURTH YEAR PIAGETIAN TASKS FROM YEAR 1 TO 4 VARIABLES, WITHOUT AND WITH AGE AS A PREDICTOR

	Standardized Regression Weights					Multiple R	F
Year 4CI		$.237CI_1$			$+ .243VM_1$	.412	13.39(2,131)
From Year 1	$-.271 A$	$+ .274CI_1$			$+ .433VM_1$	.446	10.78(3,130)
Year 4CR			$+ .179WM_1$		$+ .388VM_1$	.486	20.22(2,131)
From Year 1	$.161 A$		$+ .157WM_1$		$+ .274VM_1$	.496	14.15(3,130)
Year 4CI		$.330CI_2$			$+ .214VM_2$	.456	17.20(2,131)
From Year 2	$-.101 A$	$+ .334CI_2$			$+ .260VM_2$	.463	11.83(3,130)
Year 4CR		$.320CR_2 + .328CI_2$				.523	24.66(2,131)
From Year 2	$.224 A$	$+ .243CR_2 + .262CI_2$				.557	19.48(3,130)
Year 4CI		$.319CI_3$		$+ .246DM_3$		.451	16.72(2,131)
From Year 3	$-.011 A$	$+ .320CI_3$		$+ .250DM_3$		.451	11.07(3,130)
Year 4CR		$.374CR_3 + .189CI_3$		$+ .243DM_3$		.578	21.70(3,130)
From Year 3	$.184 A$	$+ .322CR_3 + .162CI_3$		$+ .185DM_3$		.598	17.97(4,129)
Year 4CI		$.389CR_4$	$+ .195WM_4$			.484	20.05(2,131)
From Year 4	$-.098 A$	$+ .402CR_4$	$+ .226WM_4$			.486	13.42(3,130)
Year 4CR		$.316CI_4$		$+ .211DM_4$	$+ .207VM_4$	.548	18.62(3,130)
From Year 4	$.363 A$	$+ .379CI_4$				.572	31.79(2,131)

A: Age

CR: Combinatorial Reasoning

CI: Class Inclusion

WM: Word Memory Span

DM: Digit Memory Span

VM: Visual Orientation Memory

$rACR_4 = .434$

$rACI_4 = .188$

that combinatorial reasoning is uniformly more predictable from previous years tasks than is class inclusion. However, class inclusion appears somewhat more stable, in that in all three prediction equations, earlier levels of performance on this task are significant predictors of final status, while knowledge of initial level on combinatorial reasoning does not add significantly to the prediction of final combinatorial reasoning status attainable from year one word memory span and visual memory. Further, in years two and three, class inclusion enters as a significant predictor of combinatorial reasoning, but combinatorial reasoning never predicts class inclusion. There is no support for the hypothesis that the more visual combinatorial reasoning task would be better predicted by previous levels of visual memory, while class inclusion would be better predicted by the verbal digit or word memory span tasks. Only in the concurrent year four analyses, in which the variables are of course not included as "predictors" of themselves, is there some indication that combinatorial reasoning is more related to visual memory, although each reasoning task is concurrently linked to one of the auditory memory variables. For both reasoning tasks, over the three years of predictions, predictors shift from visual memory to auditory memory as supplements of the earlier levels of the reasoning tasks themselves.

The concurrent year four regressions, suggesting a word memory span component for class inclusion and digit span and visual memory requirements for combinatorial reasoning, do serve to underscore the fact that neither task is a "pure" measure of a logical operation, and remind us that it is impossible to operationalize the measurement of an hypothesized structure

without introducing performance variance. Thus, one might agree that the relative difficulty of the two logical operations might better be determined by first predicting scores on each from a knowledge of subjects' previous and concurrent levels on perceptual tasks with which the reasoning tasks share method variance, but which are clearly not measures of the cognitive structures of interest.

The residuals from these predictions might then be thought of as measures of attainment on the logical tasks with "irrelevant" variance stripped away. This use of memory variables as covariates is reasonable if in fact they represent a capacity or substrate for the development of logical operations. If, on the other hand, the perceptual skills are to some extent resultants of developing logical operations skills, removing their apparent contribution will, if reliability of measurement is sufficient, overcorrect, taking out variance properly attributable to the logical tasks. In particular, while the year four combinatorial reasoning with clear-class inclusion correlation is .45, the partial correlation with digit memory and visual memory taken into account is .33.

Attribution of cause from correlational evidence is an area where angels traditionally fear to tread. Two approaches, both described by Kenny (1973, 1975), were applied to these data in an attempt to tease out direction of developmental influence. The first, presented in the 1973 volume, Structural Equation Models in the Social Sciences, and elaborated in a 1975 article, corrects the synchronous and lagged correlations of two variables for presumed changes in reliability over time. If a variable is more reliable at time 2, it will be easier to predict, making it look more like an effect than it should. Changes in reliability are inferred from the pattern of correlations of the variables with other variables

measured at the two points in time. The assumption is made that variables are moving in time as a rigid system, that is, that variables are neither approaching nor receding from one another in the observation space. Thus, the corrected synchronous correlations should be approximately equal if the assumptions hold. In this case, four other variables were available to obtain independent estimates of K_x, the ratio of the reliabilities of variable X at time t_2 to t_1. Tables 5, 6, and 7 show the observed and corrected correlations and corresponding values of K for the logical tasks and visual memory for all adjacent years, and for other predictors of year four logical tasks in the years in which they played significant roles in the regression equations.

Observing the adjacent-year cross-lagged correlations between visual memory and class inclusion (Table 5), we see that the "quasi stationarity" assumption of equal corrected synchronous correlations is met quite satisfactorily for the first two years, but that the corrected cross-lagged correlations are not sufficiently discrepant to lead to any inference concerning the direction of causality. From years three to four, the communality method leads to a marked estimated drop in the reliability of the memory variable and an estimated increase in the reliability of the class inclusion measure, making the corrected cross-lagged correlations more discrepant than the observed values.

The difference, suggesting a tendency for changes in class inclusion to be causally prior to changes in memory, barely fails to attain significance at the .05 level ($Z = 1.88$), but in both the years three to four and the years one to four analyses, corrected synchronous correlations may be too different (.35 versus .31, .39 versus .35) to make the hypothesis of quasi-stationarity appealing.

Turning to the cross-lagged relationships between visual

TABLE 5

CROSS-LAGGED CORRELATIONS:

VISUAL MEMORY (VM) AND CLASS INCLUSION (CI)

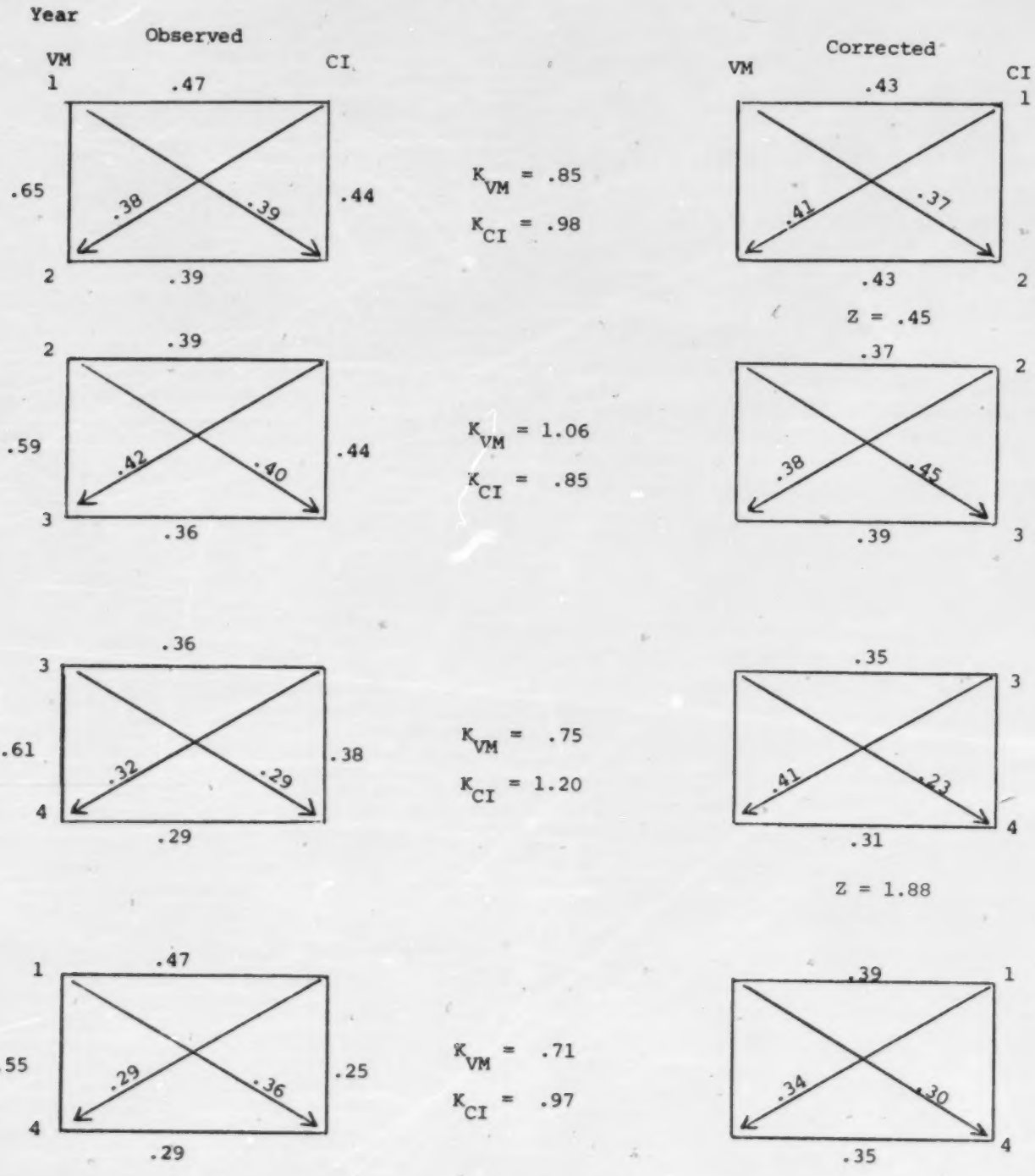


TABLE 6

CROSS-LAGGED CORRELATIONS:

VISUAL MEMORY (VM) AND COMBINATORIAL REASONING (CR)

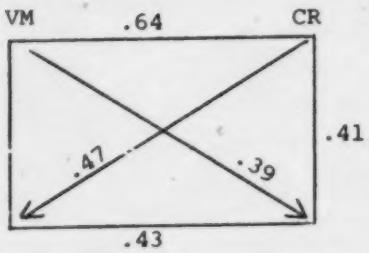
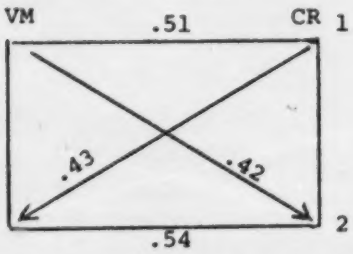
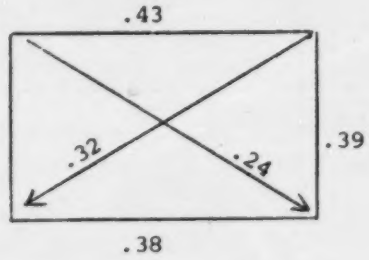
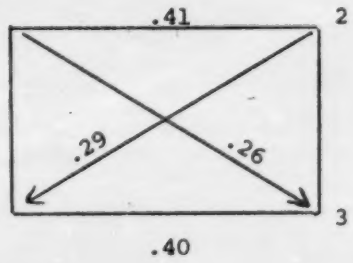
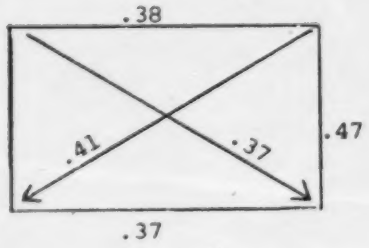
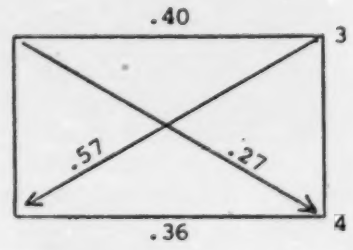
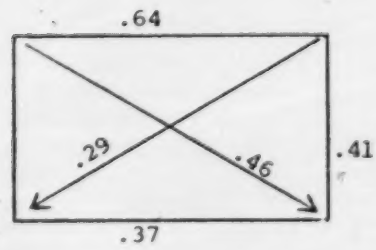
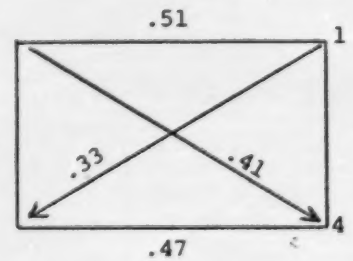
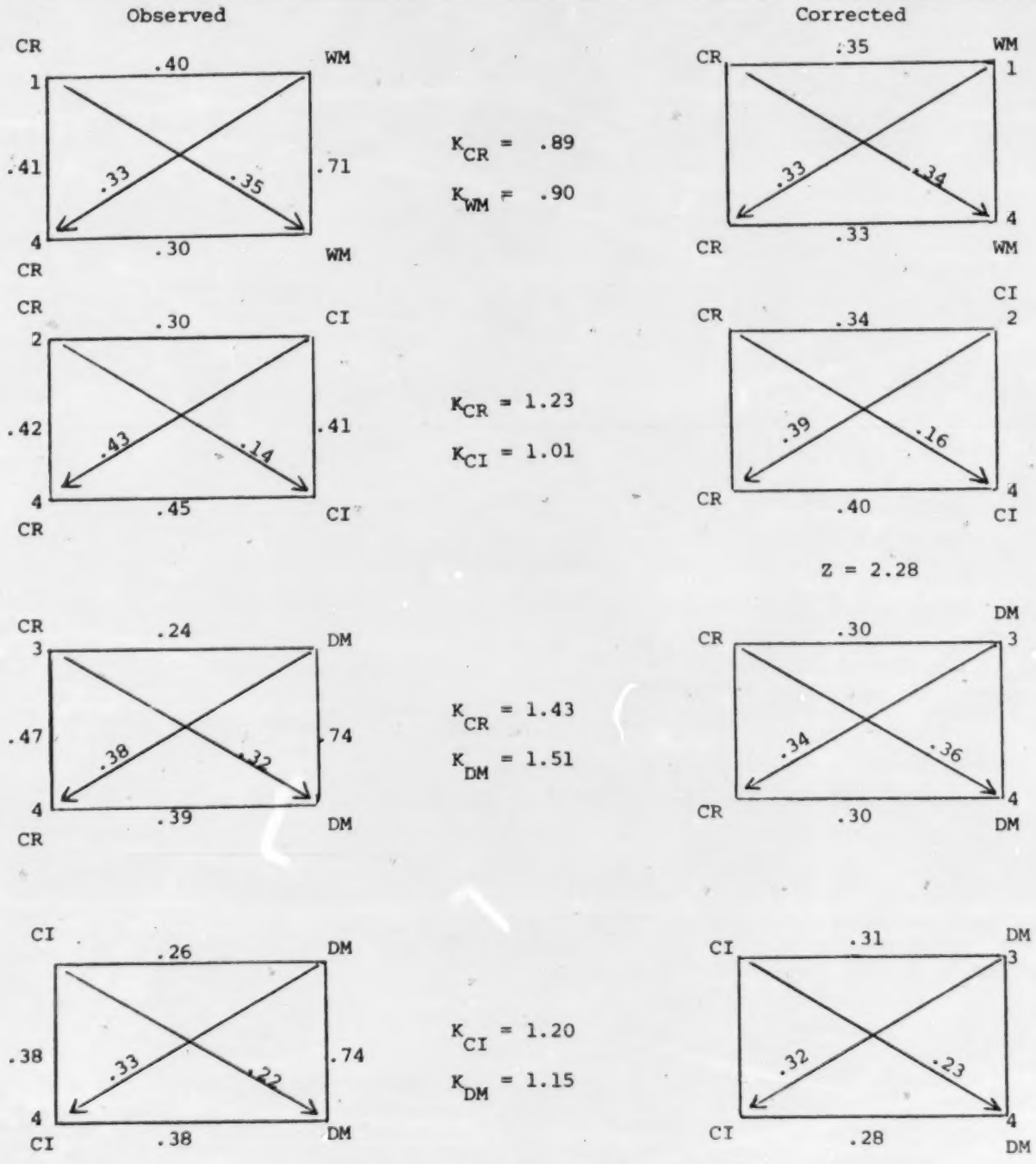
Year	Observed		Corrected
1		$K_{VM} = .85$ $K_{CR} = .73$	
2		$K_{VM} = 1.06$ $K_{CR} = .89$	
3		$K_{VM} = .75$ $K_{CR} = 1.43$	 $Z = 3.38$
1		$K_{VM} = .71$ $K_{CR} = .89$	 $Z = .84$

TABLE 7

CROSS-LAGGED CORRELATIONS:

WORD MEMORY, DIGIT MEMORY, AND THE PIAGETIAN VARIABLES



Partial Correlations Year 4

$r_{CICR \cdot VM}$ = .38 $r_{CICR \cdot WM}$ = .37

$r_{CICR \cdot VMDM}$ = .33

memory and combinatorial reasoning (Table 6), the communality method leads to reliability corrections which attenuate the observed differences in cross-lagged correlations from years one to two and from years two to three, but a sharp increase in the year four estimated reliability of combinatorial reasoning amplifies the excess of $\underline{r}_{CR_3VM_4}$ over $\underline{r}_{VM_3CR_4}$ raising the difference to a highly significant level ($\underline{Z}$ = 3.38), and suggesting that growth in combinatorial reasoning leads to growth in visual memory. Since such a developmental influence could hardly alter a fundamental physiological capacity, if we accept this finding, then the most reasonable causal link would seem to be not growth in memory space but a mediational process of change in means and efficiency of memoric encoding due to the emergence of new logical structures. However, the years one to four results, while not significant, are apparently in the opposite direction from those of each of the intervening adjacent years, suggesting that the hypothesis that change is occurring as a motion of a rigid system, with increases and decreases in correlations reflecting only changed reliabilities rather than relative directions of vectors in the system, may not be tenable.

Table 7 presents the observed and reliability-corrected cross-lagged correlations among year four class inclusion and combinatorial performances and those remaining previous years' variables which entered significantly into the regression equations. The only significant difference among cross lags, $\underline{Z}$ = 2.28, occurs for year two class inclusion as a predictor of year four combinatorial reasoning. Again, however, the corrected synchronous correlations, .34 and .40, are sufficiently disparate to call into question the assumption on which the reliability corrections are based, although again, not significantly different with this sample size.

An alternative test offered in Kenny (1973) is based on the availability of additional occasions of measurement rather than of additional variables measured at two occasions. In this approach, reliabilities are not directly manipulated, but loadings on a postulated synchronous common factor and on a lagged factor are assumed to stay in a constant proportion. That is, observation vectors may converge or diverge over time, but they are assumed to do so at a steady rate. Kenny offers three wave causal contrasts in two forms: one form involving synchronous correlations and one only lagged correlations. It is easy to generalize these equations for the solution of four-wave data, and this has been done in Table 8. The first two columns represent the values of the two components of the test based on synchronous correlations. If the value of the left column is significantly more positive than that of the middle column, cause is presumed to flow from x to y. The discrepancy is greatest for CR to VM and in the other direction, from CI to CR, supporting the findings obtained through reliability correction.

However, the fact that small negative values appear for at least one component of each contrast suggests that the synchronous correlations rx_2y_2 and rx_3y_3 are slightly inflated by shared method variance. To the extent that the negative values are not just due to sampling error, the assumptions of this test can also be questioned. The last column of Table 8, however, offers a test which does not depend on synchronous correlations, being affected by only the factors hypothesized to influence time-lagged correlations. This test in fact does not depend on synchronous data at all, and again leads to the conclusion that combinatorial reasoning exerts a causal influence on the development of visual memory and that class inclusion is developmentally prior to combinatorial reasoning.

TABLE 8*

FOUR-WAVE CAUSAL EQUATIONS

X	Y	$rx_1y_2rx_2y_3rx_3y_4$ $- rx_2y_2rx_3y_3rx_1y_4$	$rx_2y_1rx_3y_2rx_4y_3$ $- rx_2y_2rx_3y_3rx_4y_1$	$\frac{rx_1y_2rx_2y_3rx_3y_4}{rx_1y_4}$ $- \frac{rx_2y_1rx_3y_2rx_4y_3}{rx_4y_1}$
CR	VM	.014	-.041	.137
CR	WM	-.008	-.020	.036
CR	DM	-.020	-.012	.001
CR	CI	-.001	.026	-.118
CI	VM	.006	-.006	.038
CI	WM	-.015	-.004	-.015
CI	DM	-.017	.001	-.052

*see Appendix

Partial correlation, removing covariance predictable from initial status on memory variables, considerably reduced the relationship between the two Piagetian tasks in the final year ($r_{CICR} = .45$, $r_{CICR \cdot VMDM} = .33$), suggesting that short-term memory factors account for a significant portion of the covariance of the two logical operations tasks. Examination of the cross-lagged panel correlations, however, suggests that using the memory variables as "passive" covariates may represent an oversimplification. In each of the 3 adjacent year comparisons, previous level of combinatorial reasoning predicts visual memory at a slightly higher level than visual memory predicts combinatorial reasoning (r = .47 vs .39, .32 vs .24, .41 vs .37). This suggests that visual memory task performance is not merely a background or capacity variable, but may itself be influenced by emerging operational structures. Class inclusion and digit span reveal an inconsistent pattern of direction of influence.

These results make it clear that individual differences in short-term memory development contribute to the relationships of these concrete operations performances. However, the direction of apparent causality between combinatorial reasoning and visual memory in all years, and with auditory memory from years two to three, suggests that changes in memory performance may also be to some degree a <u>result</u> of the development of cognitive operations. To the extent that the assumptions of these tests are reasonable, the use of memory variables as covariates will be counterindicated, and the development of reciprocal causal models will be necessary to describe these relationships. Recent discussions of the complex interrelationship among memoric processes, metamemory, and cognitive developmental variables support this conclusion (Flavell, 1977; Hagen, Jongeward, & Kail, 1975; and Kail

& Hagen, 1976).

That class inclusion influences the emergence of combinatorial reasoning is not surprising. Class inclusion, as conceptualized and measured by Piaget, is one of the more difficult attainments of the concrete operations period, but combinatorial reasoning is necessary to formal operations. The finding of a causal link from combinatorial reasoning to visual memory may be less in accord with intuition or conceptual task analyses. Since McLaughlin (1963), information processing theorists have thought of short term memory as a capacity or working space limiting the complexity of cognitive operations. However, more recent approaches to memory development (for example, see Flavell, 1977, pages 183-218) have emphasized encoding, structuring and retrieval processes and strategies as the entities in which development works its changes, rather than seeing development as the physiological growth of computing space.

This emphasis on the dynamic structuring of input rather than its passive filing and retrieval is of course consistent with Piaget's views, but has unfortunate implications for the possibility of simply partialling out memory factors to lay bare the outline of logical structure. Having set out to weave a comfortable rug of covariance from which to contemplate the development of logical operations, we seem to have managed to pull the rug out from under ourselves, to land with a jolt on the rough surface of non-recursive structural equation models.

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Appendix

The Cross-lagged Panel Approach (Campbell, 1963; Campbell & Stanley, 1963) represents an attempt to adduce evidence of causation from longitudinal correlational data. In its simplest, two wave, two variable embodiment, two variables, X and Y are observed at two points in time, 1 and 2. The resulting model of causal linkages leading to X_2 and Y_2 may be expressed as a path model (Wright, 1960):

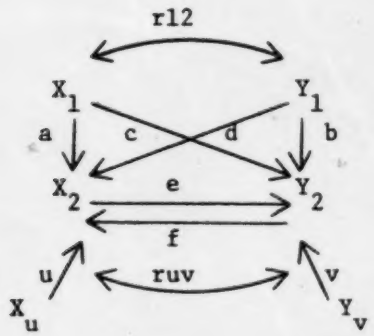
$$X_2 = aX_1 + CY_1 + eY_2 + \mu X_u$$

$$Y_2 = bY_1 + dX_1 + fX_2 + vY_v$$

where X_u and Y_v are residuals for X_2 and Y_2, respectively.

This model may be represented as in Figure 1.

Figure 1



Unfortunately, this general model is underidentified. In the absence of data from other concurrent variables or from other time waves, stringent assumptions must be made if causality is to be inferred from the patterns of correlations (Duncan, 1969). In Rozelle and Campbell's (1969) illustration of the model, the time lag is assumed short enough to preclude "instantaneous"

causation, and e and f are hence set to zero. If the causal relationships between X and Y are stationary, implying that $\rho X_1 Y_1 \sim \rho X_2 Y_2$, then a positive cross-lagged differential. $\rho X_1 X_2 - \rho X_2 Y_1$ indicates that X is the predominant cause of Y over the observed time interval. The null hypothesis is that a third variable or factor causes both X and Y with identical time lags.

In many applications, the apparent relationships between X and Y change over time, in violation of the stationarity requirement. Two hypotheses for this situation: changes in the reliability of the variables, with underlying stationarity of true scores; or change in the causal structure of variables, with constant reliabilities, have been proposed and solved by Kenny (1973, 1975).

In the case of hypothesized changes in reliability, Kenny forms a set of q_{ij}, where q_{ij} is the ratio of synchronous correlations of variables i and j at time two and time one. For example for years three and four,

$$q_{VW} = \frac{.251}{.369} \quad q_{VD} = \frac{.348}{.365} \quad q_{VR} = \frac{.373}{.383} \quad q_{VI} = \frac{.293}{.364} \quad q_{WD} = \frac{.688}{.743}$$

$$q_{RD} = \frac{.389}{.237} \quad q_{RI} = \frac{.447}{.224} \quad q_{RW} = \frac{.296}{.301} \quad q_{DI} = \frac{.334}{.264} \quad q_{WI} = \frac{.310}{.336}$$

(V = VM, D = VD, R = CR, I = CI, W = WM; $q_{ij} = q_{ji}$)

Then 6 estimates of the reliability ratio, or communality, K_i^2, of VM and CR from years three to four may be obtained, e.g.:

$$K_v^2 = \frac{q_{VW}\, q_{VD}}{q_{WD}} = \frac{(.6802)(.9534)}{.9260} = .700$$

$$K_v^2 = \frac{q_{VI}\, q_{VD}}{q_{ID}} = \frac{(.8049)(.9534)}{1.2652} = .607$$

$$K_R^2 = \frac{q_{RW}\, q_{RD}}{q_{WD}} = \frac{(.9834)(1.6414)}{.9260} = .1.743$$

$$K_R^2 = \frac{q_{RV}\, q_{RI}}{q_{VI}} = \frac{(.9739)(1.9955)}{.8049} = 2.414$$

$$K_V^2 = \frac{q_{VR}\, q_{VW}}{q_{RW}} = \text{etc.}$$

Kenny suggests pooling these estimates by adding numerators and denominators separately, yielding communalities:

$$\hat{K}_v^2 = \frac{4.338}{7.734} = .5609$$

$$\hat{K}_v = .7489$$

$$\hat{K}_R^2 = \frac{11.3517}{5.523} = 2.045$$

$$\hat{K}_R = 1.430$$

Thus the reliability of VM appears to have decreased from year three to year four, while the reliability of CR appears to have increased. By analogy to correction for attenuation, we form $\sqrt{\frac{K_v}{K_R}} = .7237$, $\sqrt{K_v K_R} = 1.0348$

$$\sqrt{\frac{K_R}{K_v}} = 1.3813, \quad \frac{1}{\sqrt{K_v K_R}} = .9664$$

The observed cross-lagged correlations are corrected as follows:

$$r'_{VM_3CR_3} = \sqrt{\frac{K_v}{K_R}}\ r_{VM_3\,CR_4} = (.7237)(.37) = .27$$

$$r'_{CR_3VM_4} = \sqrt{\frac{K_R}{K_V}}\ r_{CR_3VM_4} = (1.3813)(.41) = .57$$

The synchronous correlations are also corrected:

$$r'_{VM_3CR_3} = \sqrt{K_V K_R}\ r_{VM_3CR_3} = (1.0348)(.38) = .40$$

$$r'_{VM_4CR_4} = \sqrt{\frac{1}{K_V K_R}}\ r_{VM_4CR_4} = (.9664)(.37) = .36$$

Corrected cross-lagged correlations are now tested by means of test attributed to Pearson and Filon (Kenny, 1975, p. 896): form k =

$$(r_{12} - r_{24}r_{14})(r_{34} - r_{24}r_{23}) + (r_{13} - r_{12}r_{23})(r_{24} - r_{12}r_{14}) +$$

$$(r_{12} - r_{13}r_{23})(r_{34} - r_{13}r_{14}) + (r_{13} - r_{14}r_{34})(r_{24} - r_{34}r_{23})$$

$$\text{Then } Z = \frac{\sqrt{N(r_{14} - r_{23})}}{\sqrt{(1 - r_{14}^2)^2 + (1 - r_{23}^2)^2 - k}}$$

where N is the sample size, is normally distributed.*

Kenny's alternative (1973) approaches lead to the results in Table 8 of the report. Extending his model for three-wave two-variable data to four-wave 2-V data is straightforward, as follows.

Assume that the reliability of the two variables X and Y does not change over the four occasions, but that their dependence on a common factor Z may change over time. The null hypothesis, in terms of X_i, Y_i, Z_i, and specific variances U_i and V_i, and errors e_i, of prediction of the factor from its previous values, can be expressed in the following model:

---

*Thus, for the years three and four example,

$$K = (.396 - (.474)(.266))\ (.360 - (.474)(.567)) +$$
$$(.613 - (.396)(.567))\ (.474 - (.396)(.613)) +$$
$$(.396 - (.613)(.567))\ (.360 - (.613)(.266)) +$$
$$(.613 - (.266)(.360))\ (.474 - (.360)(.567)) = .2635$$

$$Z = \frac{\sqrt{134}(.567 - .266)}{\sqrt{(1 - .266^2)^2 + (1 - .567^2)^2 - .2635}} = 3.3837$$

These calculations for Tables 5-7 were performed from the correlations with the aid of an SR-51 calculator and several pots of Medaglio d'Oro coffee.

$$X_1 = a_1Z_1 + U_1 \qquad Y_1 = b_1Z_1 + V_1$$

$$X_2 = a_2Z_2 + U_2 \qquad Y_2 = b_2Z_2 + V_2$$

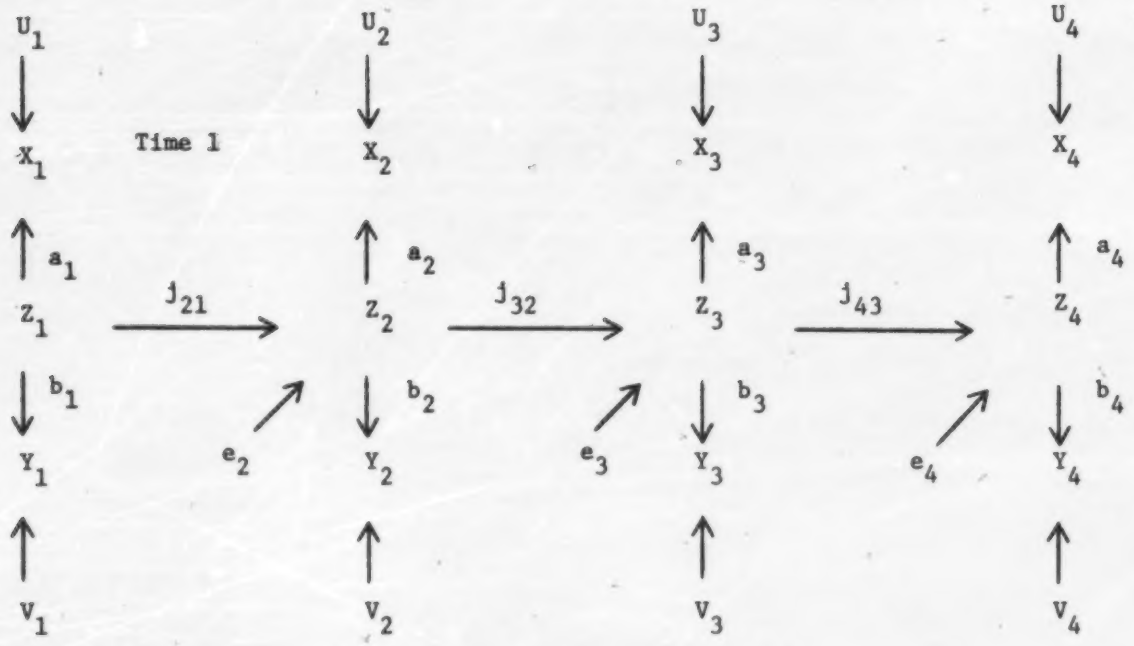
$$X_3 = a_3Z_3 + U_3 \qquad Y_3 = b_3Z_3 + V_3$$

$$X_4 = a_4Z_4 + U_4 \qquad Y_4 = b_4Z_4 + V_4$$

$$Z_2 = j_{21}Z_1 + e_2$$

$$Z_3 = j_{32}Z_2 + e_3 = j_{32}(j_{21}Z_1 + e_2) + e_3$$

$$Z_4 = j_{43}Z_3 + e_4 = j_{43}(j_{32}Z_2 + e_3) + e_4 = j_{43}(j_{32}(j_{21}Z_1 + e_2) + e_3) + e_4$$



Under the null hypothesis of no direct causal linkage between X's and Y's, the following path equations may be written:

$\rho X_1Y_2 = a_1 j_{21} b_2$ $\quad$ $\rho X_1Y_4 = a_1 b_4 j_{21} j_{32} j_{43}$ $\quad$ $\rho X_1Y_1 = a_1 b_1$

$\rho X_2Y_3 = a_1 j_{21} j_{32} b_3$ $\quad$ $\rho X_2Y_4 = a_2 b_4 j_{32} j_{43}$ $\quad$ $\rho X_2Y_2 = a_2 b_2$

$\rho X_2Y_1 = a_2 j_{21} b_1$ $\quad$ $\rho X_3Y_4 = a_3 b_4 j_{43}$ $\quad$ $\rho X_3Y_3 = a_3 b_3$

$\rho X_3Y_1 = a_3 j_{32} j_{21} b_1$ $\quad$ $\rho X_4Y_1 = a_4 b_1 j_{21} j_{32} j_{43}$ $\quad$ $\rho X_4Y_4 = a_4 b_4$

$\rho X_3Y_2 = a_3 J_{32} b_2$ $\quad$ $\rho X_4Y_2 = a_4 b_2 j_{32} j_{43}$

$\rho X_4Y_3 = a_4 b_3 j_{43}$

This leads to independent estimates of j_{21}, j_{32}, and j_{43} which under the hypothesis should be equal, e.g.:

$$1 \qquad j_{21}^2 = \frac{(a_1 b_4 j_{21} j_{32} j_{43})(a_2 b_1 j_{21})}{(a_2 b_4 j_{32} j_{43})(a_1 b_1)} = \frac{\rho X_1Y_4 \rho X_2Y_1}{\rho X_2Y_4 \rho X_1Y_1}$$

$$2 \qquad = \frac{(a_1 b_2 j_{21})(a_2 b_1 j_{21})}{(a_1 b_1)(a_2 b_2)} = \frac{\rho X_1Y_2 \rho X_2Y_1}{\rho X_1Y_1 \rho X_2Y_2}$$

$$3 \qquad j_{32}^2 = \frac{(a_2 b_4 j_{43} j_{32})(a_3 b_2 j_{32})}{(a_3 b_4 j_{43})(a_2 b_2)} = \frac{\rho X_2Y_4 \rho X_3Y_2}{\rho X_3Y_4 \rho X_2Y_2}$$

$$4 \qquad = \frac{(a_2 b_3 j_{32})(a_3 b_2 j_{32})}{(a_2 b_2)(a_3 b_3)} = \frac{\rho X_2Y_3 \rho X_3Y_2}{\rho X_2Y_2 \rho X_3Y_3}$$

Then, from (1) and (2): $\frac{\rho X_1Y_4}{\rho X_2Y_4} = \frac{\rho X_1Y_2}{\rho X_2Y_2}$ and from (3) and (4): $\frac{\rho X_2Y_4}{\rho X_3Y_4} = \frac{\rho X_2Y_3}{\rho X_3Y_3}$

multiplying left and right members of the equalities, $\frac{\rho X_1Y_4}{\rho X_3Y_4} = \frac{\rho X_1Y_2 \rho X_2Y_3}{\rho X_2Y_2 \rho X_3Y_3}$ or $\rho X_1Y_2 \rho X_2Y_3 \rho X_3Y_4 - 5X_2Y_2 \rho X_3Y_3 \rho X_1Y_4 = 0$, and if the expression is sufficiently positive, the null hypothesis is rejected and causation tends to flow from X to Y. The second column of Table 6 is derived similarly. If this expression is positive, causation tends to flow from Y to X. If one

expressions is essentially zero, the model is not contradicted. However significant negative values would suggest that synchronous correlations are inflated, and that the autocorrelations (j_{ik}) have been underestimated. The distribution of these statistics has not, however been obtained (Kenny, personal communication, 1975).

The final column of Table 6 represents an alternative test which does not depend on synchronous correlations. This is obtained from the above model as follows:

$$\frac{\rho X_1 Y_2 \rho X_2 Y_3 \rho X_3 Y_4}{\rho X_1 Y_4} = \frac{a_1 j_{21} b_2 a_2 j_{32} b_3 a_3 j_{43} b_4}{a_1 j_{21} j_{32} j_{43} b_4} = a_2 b_2 a_3 b_3$$

$$\frac{\rho X_2 Y_1 \rho X_3 Y_2 \rho X_4 Y_3}{\rho X_4 Y_1} = \frac{a_2 j_{21} b_1 a_3 j_{32} b_2 a_4 j_{43} b_3}{a_4 j_{43} j_{32} j_{21} b_1} = a_2 b_2 a_3 b_3$$

Thus, under the null hypothesis,

$$\frac{r_{X_1 Y_2} r_{X_2 Y_3} r_{X_3 Y_4}}{r_{X_1 Y_4}} - \frac{r_{X_2 Y_1} r_{X_3 Y_2} r_{X_4 Y_3}}{r_{X_4 Y_1}} = 0,$$

While if this difference is positive, causation is interpreted as folowing predominantly from the X to the Y variable over the three year time interval.

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Marvin J. Fruth
Area Chairperson
Studies of Implementation of Individualized Schooling

Herbert J. Klausmeier
Area Chairperson
Studies of Instructional Programming for the Individual Student

James M. Lipham
Area Chairperson
Studies of Administration and Organization for Instruction

Thomas A. Romberg
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